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Dom-Total Chromatic Number of Some Graphs

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ABSTRACT: Let G be a graph. For a given χ - colouring of a graph G, a dominating set S ⊆ V (G) is said to be a dom-colouring set if it contains atleast one vertex of each colour class of G. The dom -total chromatic number of a graph G is the minimal cardinality taken over all its dom- total colouring sets and is denoted by $\gamma_{d\text{-}tc}$ (G). In this paper, we introduce algorithms to obtain the dom-total colouring and dom-total chromatic number of various classes of graphs.

KEYWORDS: Dom- chromatic number, Dom- colouring set, Dom-total chromatic number, gear graph, pan graph, sunlet graph, web graph.

I. INTRODUCTION

All graphs considered in this paper are finite, undirected graphs and we follow standard definition of graph theory as found in [1]. Let G = (V, E) be a graph. The open neighborhood N(v) of a vertex v ∈V(G) consists of the set of all vertices adjacent to v. The closed neighborhood of v is N[v] = N (v)∪{v}. An induced subgraph G[S], where S of a graph G is a graph formed from a subset S of the vertices of G and all of the edges connecting pairs of vertices in S. A graph in which every pair of vertices is joined by exactly one edge is called complete graph. A complete bi partite graph is a graph whose vertices can be partitioned into two subsets $V_1$and V_2 such that no edge has both end points in the same subset, and each vertex of V_1 is connected to every vertex of V_2 and vice -verse. A star graph S_n is the complete bipartite graph $K_{1,n-1}$ (A tree with one internal node and n-1leaves).

The path and cycle of order n are denoted by P_n and C_n respectively. For any two graphs G and H, we define the cartesian product, denoted by G×H, to be the graph with vertex set V(G)×V(H) and edges between two vertices (u_1,v_1) and (u_2,v_2) iff either $u_1=u_2$ and $v_1v_2 \in E$ (H) or $u_1u_2 \in E(G)$ and $v_1=v_2$.

A subset S of V is called a dominating set if every vertex in V-S is adjacent to atleast one vertex in S. The dominating set is minimal dominating set if no proper subset of S is a dominating set of G. The domination number γ is the minimum cardinality taken over all minimal dominating set of G. A γ-set is any minimal dominating set with cardinality γ.

A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The minimum number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by χ (G). A dominator coloring of G is a proper coloring of G in which every vertex of G dominates atleast one-color class. The dominator chromatic number is denoted by χ_d (G) and is defined by the minimum number of colors needed in a dominator coloring of G. This concept was introduced by Ralucca Michelle Gera in 2006[2].

In a proper coloring C of G, a color class of C is a set consisting of all those vertices assigned the same color. Let C^1 be a minimal dominator coloring of G. We say that a color class $c_i \in C^1$ is called a non-dominated color class (n-d color class) if it is not dominated by any vertex of G. These color classes are also called repeated color classes.

A total coloring of a graph G is a mapping $f: V(G) \cup E(G) \rightarrow C$, where C is set of colors which satisfy the condition

i) No two adjacent vertices receive the same color.

ii) No two adjacent edges receive the same color.

iii)No edges and its end vertices receive the same color.

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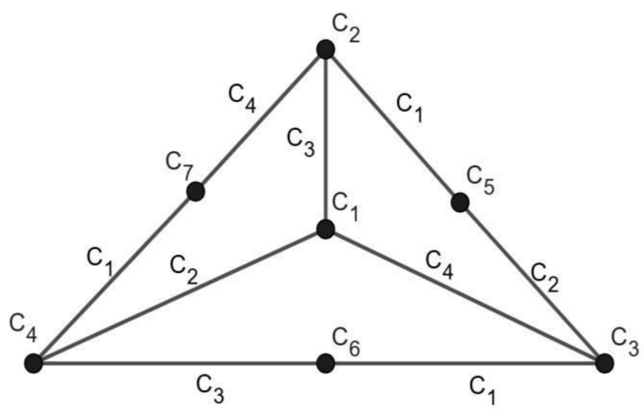
The Total chromatic number $\chi_T(G)$ of a graph G is the minimum number of colors required for a total coloring of a graph G.

Let $G = (V, E)$ be a finite simple graph with total chromatic number $\chi_T(G)$ and domination number $\gamma(G)$. A Dom-Total Chromatic Number of G is denoted by $\gamma_{d-tc}(G)$, is defined as $\gamma_{d-tc}(G) = \chi_T(G) - \gamma(G)$. For a given χ - total colouring of a graph G, a dominating set S ⊆ V (G) is said to be a dom-total colouring set if it contains atleast one vertex of each colour class of G. The dom chromatic number of a graph G is the minimal cardinality taken over all its dom-total colouring sets and is denoted by γ_{d-tc} (G). A Gear graph G_n is the graph obtained from the cycle C_n by inserting one new vertex into every edge of the cycle and then adding a central vertex that is adjacent to each of the original cycle vertices. The n-pan graph, denoted by Pn_n is the graph obtained by joining a cycle graph C_n and a singleton graph K_1 by a bridge. i.e., Pn_n consists of a cycle on n-vertices together with an additional vertex connected to exactly one vertex of the cycle by a single edge. The n-sunlet graph, denoted by S_n, is a graph on $2n$ vertices obtained by attaching n-pendant vertices to the cycle graph C_n. The Web graph Wb_n is defined as a stacked prism graph $Y_{n+1,3}$ with the edges of the outer cycle removed. Web graph has $3n$ vertices and $4n$ edges.

II. MAIN RESULTS

Theorem :1
The Dom-Total chromatic number of Gear graph is $2n - 1$.
i.e. $\gamma_{d-tc}(G_n) = 2n - 1, n \geq 3$.
Proof: Let $G = G_n$ be the Gear graph with the vertex set $(G_n) = \{v_1, v_2, v_3, \cdots, v_{2n}, v_{2n+1}\}$
Let the set of colors be $C = \{c_1, c_2, c_3, \cdots, c_{2n+1}\}$. Assign the color c_1 to the root vertex. The outer vertices of the cycle are colored with the colors $\{c_1, c_2, c_3, \cdots, c_{n+1}\}$.The intermediate vertices $\{v_{i,i+1}, v_{i+1,i+2}, \cdots, v_{n,i}\}$ of the outer cycle are assigned with different colors $\{c_{n+2}, c_{n+3}, \cdots, c_{2n+1}\}$.The outer edges colored alternatively with the colors c_1, c_2 and c_1, c_3 ,⋯ and c_1, c_{n+1}. Finally, the edge incident with root vertex is colored with the colors $\{3, \cdots, c_{n+1}, c_2\}$
The number of colors required for the total coloring of G_n is $2n + 1$. Hence, the total chromatic number of G_n is $2n + 1$ .i.e., $\chi(G_n) = 2n + 1$, $n \geq 3$
The minimum dominating set of Gear graph G_n is 2. We have, $\gamma_{d-tc}(G_n) = 2n + 1 - 2 = 2n - 1$
Hence, Dom-Total Chromatic number of Gear graph G_n is $2n - 1$. $\gamma_{d-t}(G_n) = 2n - 1, n \geq 3$.

Illustration :1,2



Total coloring of Gear graph G_3 and $\gamma_{d-tc}(G_3) = 5$

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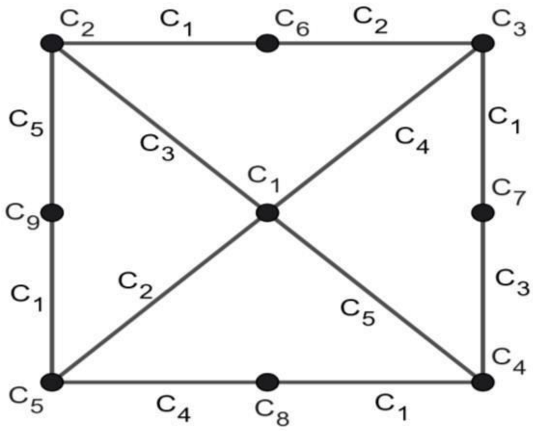

Total coloring of Gear graph G_4 and γ_{d-tc} $(G_4) = 7$

Theorem :2
The Dom-Total Chromatic Number of Pan graph Pn_n is n
i.e., γ_{d-tc} $(Pn_n) = n,\ n \geq 3$
Proof:
Let $G = Pn_n$ be the pan graph. Let $\{v_1, v_2, \cdots, v_n\}$ be outer vertices of the cycle graph C_n. Let $\{e_1, e_2, \cdots, e_n\}$ be edges of the cycle graph C_n .
Let f_1 be the bridge connecting a vertex of C_n to a singleton vertex of K_1 .
Let the set of colors be $\{c_1, c_2, \cdots, c_{n+1}\}$.Now the coloring assignment of Pn_n is given below
The edges $\{e_1, e_2, \cdots, e_n\}$ are colored with the colors $\{c_2, c_3, \cdots, c_{n+1}\}$ (anticlockwise). The vertices $\{v_1, v_2, \cdots, v_n\}$ of the cycle graph are colored with the colors $\{c_1, c_2, \cdots, c_n\}$ (anticlockwise)
The bridge f_1 is colored with the color c_1.The vertex K_1 is colored with the color

The number of colors required for the total coloring of Pn_n is $n + 1$. Hence, the total chromatic number of Pn_n is $n + 1$. i.e., $\chi_T(Pn_n) = n + 1,\ n \geq 3$
The minimum dominating set of Pan graph is 1. We have, $\gamma_{d-tc}(Pn_n) = n + 1 - 1 = n$
Hence, Dom-Total Chromatic number of Pan graph Pn_n is .
$\gamma_{d-t}(Pn_n) = n\ ,\ n \geq 3.$

Illustration :1

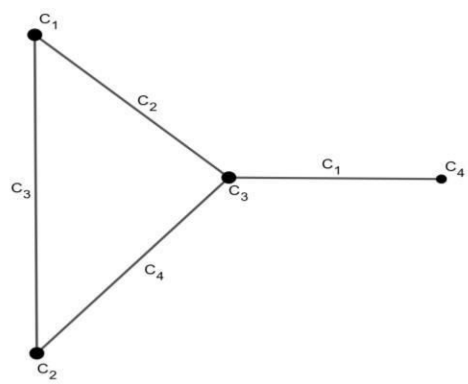

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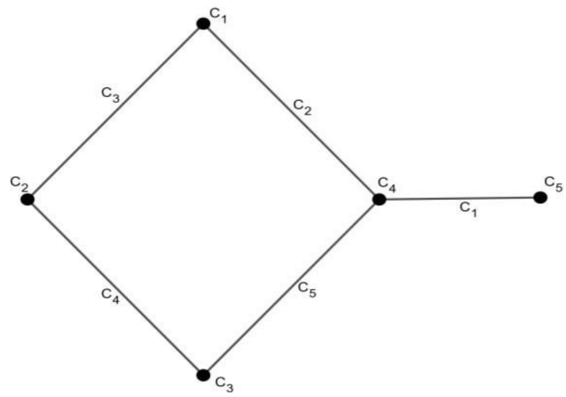
Illustration:2

Total coloring of Pan graph Pn_3 and γ_{d-tc} (Pn_3) = 3



Total coloring of Pan graph Pn_4 and γ_{d-tc} (Pn_4) = 4.

Theorem :3

The Dom-Total Chromatic Number of Sunlet graph S_n

i.e., $\gamma d{-}tc$

Proof:

(S_n)

$= n + 2 - [\frac{n}{2}], \ \forall\, n \geq 5$

except when n=3 and 4.

Let $G = S_n$ be the Sunlet graph.Let $\{v_1, v_2, \cdots, v_n\}$ be the vertices of the cycle graph C_n and let $\{u_1, u_2, \cdots, u_n\}$ be the pendant vertices. Let $\{e_1, e_2, \cdots, e_n\}$ be the edges of the cycle graph C_n and $\{f_1, f_2, \cdots, f_n\}$ be the pendant edges.

Now the coloring assignment of S_n is given below.The vertices $\{v_1, v_2, \cdots, v_n\}$ of the cycle graph C_n are colored with the colors $\{c_1, c_2, \cdots, c_n\}$[clockwise] and the edges $\{e_1, e_2, \cdots, e_n\}$ are colored with the colors $\{c_n, c_{n+1}, \cdots, c_2\}$.

The pendant vertices $\{u_1, u_2, \cdots, u_n\}$ are colored with the color c_{n+2}.

The edges $\{f_2, f_3, \cdots, f_n\}$ are colored with the color c_1 and the edge f_1 is colored with the color c_{n+1}.

$$(v_i) = i\,,\ 1 \leq i \leq n$$
$$c(u_i) = n + 2,\ 1 \leq i \leq n$$
$$c(e_i) = i + 2,\ 1 \leq i \leq n - 1$$
$$c(e_n) = 2$$
$$(f_i) = 1\,,\ 2 \leq i \leq n$$
$$c(f_1) = n + 1$$

The number of colors is required for the total coloring of S_n is $n + 2$. Hence, the total chromatic number of S_n is $n + 2$.

i.e., $\chi_T(S_n) = n + 2,\ \forall\, n \geq 3$

The minimum dominating set of Sunlet graph S_n

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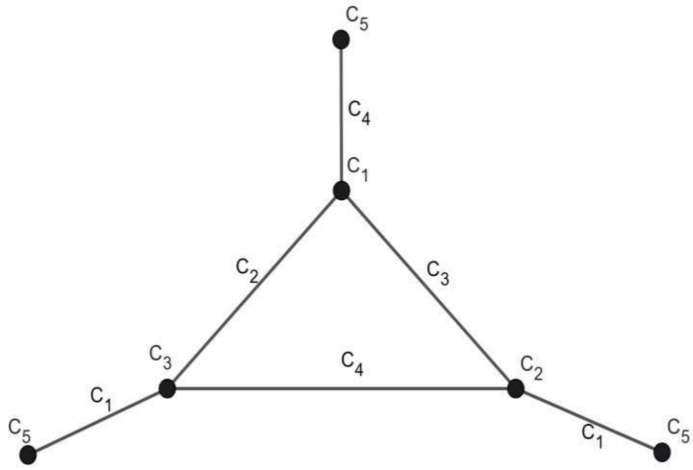
is [–] , $\forall\, n \geq 5$
2
We have, γ_{d-tc}
(S_n)
n
$= n + 2 - [–]$, $\forall\, n \geq 5$
2
Hence, Dom-Total Chromatic number of Sunlet graph S_n

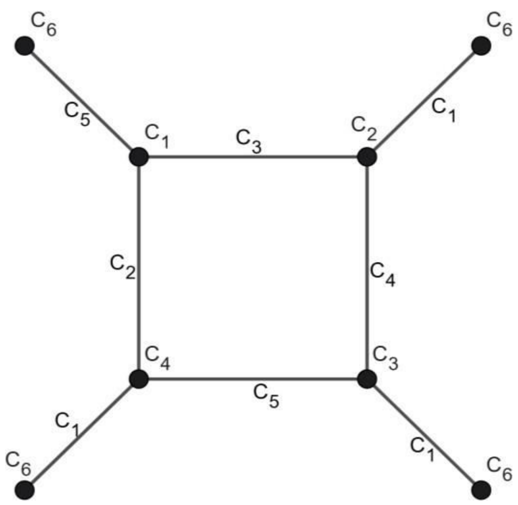
is $n + 2 - [–]$, $\forall\, n \geq 5$
2

Illustration :1



Dom-Total coloring of Sunlet graph S_3 and $\gamma_{d-tc}(S_3) = 2$

Illustration :2



Dom=Total coloring of Sunlet graph S_4 and $\gamma_{d-tc}(S_4) = 2$

Theorem :4
The Dom-Total Chromatic Number of Web graph Wb_n is $n + 1$. i.e., $\gamma_{d-tc}(Wb_n) = n + 1$, $\forall\, n \geq 3$.

Proof:
Let $G = Wb_n$ be the web graph. The coloring assignment of Wb_n is as follows.
The inner vertices of the concentric cycle graph is colored with the colors $\{c_1, c_2, \cdots, c_n\}$

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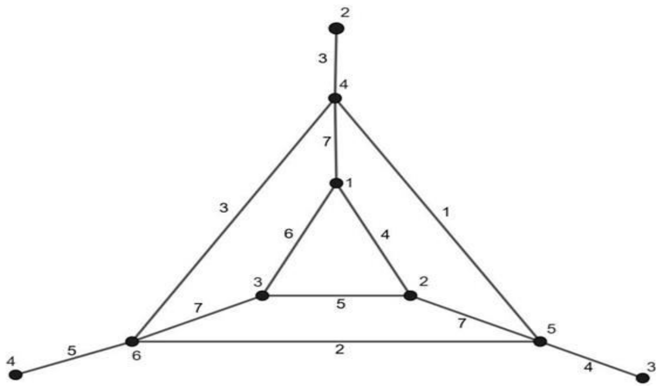
[clockwise].The outer vertices of the concentric cycle graph is colored with the colors $\{c_{n+1}, c_{n+2}, \cdots, c_{2n}\}$[clockwise].z The inner edges of the concentric cycle graph is colored with the colors $\{c_{n+1}, c_{n+2}, \cdots, c_{2n}\}$ [clockwise for $n \geq 3$, anticlockwise for $n > 3$].
The outer edges of the concentric cycle graph is colored with the colors $\{c_1, c_2, \cdots, c_n\}$ [clockwise for $n = 3$, anticlockwise for $n > 3$] .
The pendant vertices are colored with the colors $\{c_2, c_3, \cdots, c_{n+1}\}$. The pendant edges are colored with the colors $\{c_3, c_4, \cdots, c_{n+2}\}$.
The edge connecting the inner and the respective vertices of the concentric cycle are denoted by $\{e_1, e_2, \cdots, e_n\}$ and are colored with the color c_{2n+1} .
The number of colors required for the total coloring of Wb_n is $2n + 1$. Hence, the total chromatic number of Wb_n is $2n + 1$.
i.e., $\chi_T(Wb_n) = 2n + 1, \forall n \geq 3$
The minimum dominating set of Web graph Wb_n is . We have,$_{d-tc}(Wb_n) = 2n + 1 - n = n + 1$.
Hence, Dom-Total Chromatic Number of Web graph is $n + 1, n \geq 3$.

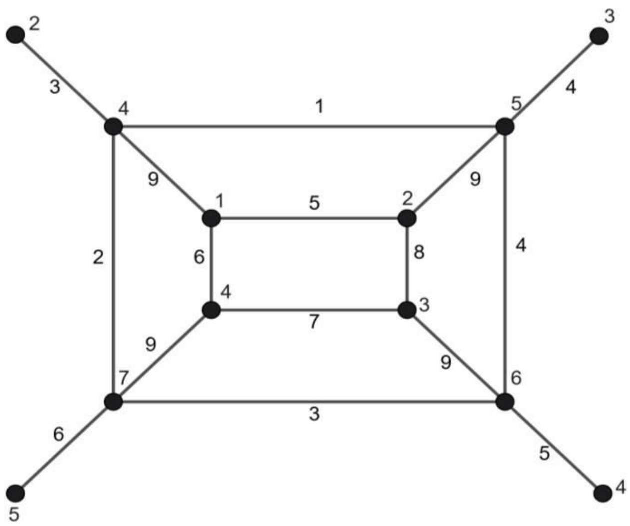
$\gamma_{d-tc}(Wb_n) = n + 1, \forall n \geq 3.$

Illustration :1



Dom-Total coloring of web graph Wb_3 and $\gamma_{d-tc}(Wb_3) = 4$

Illustration :2



Dom-Total coloring of web graph Wb_4 and $\gamma_{d-tc}(Wb_4) = 5$

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